LearnoHub
learning simplified

JEE - Mathematics

www.learnohub.com

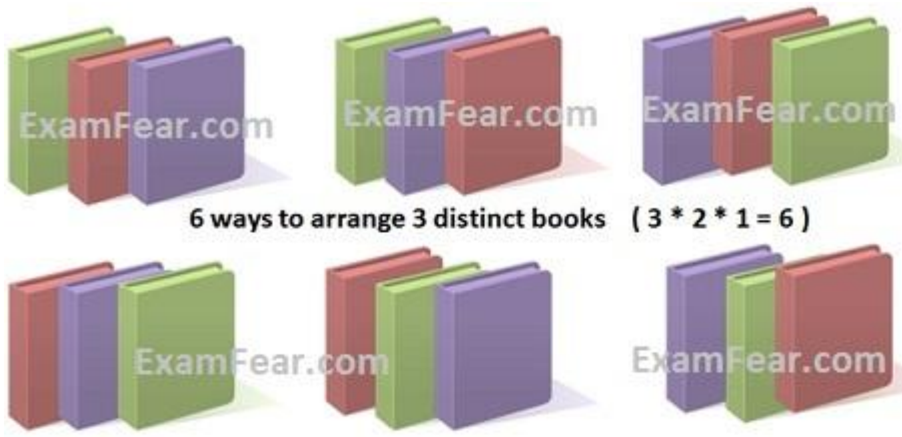
Permutations and Combinations

Revise Notes

Permutation

Permutation is an arrangement in a definite order of things.

Eg: Arrange 3 distinct books: Red, Blue & Green books.



In how many ways we can arrange (1,2,3,4)?

1 2 3 4	2 1 3 4	3 1 2 4	4 1 2 3
1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
1 3 2 4	2 3 1 4	3 2 1 4	4 2 1 3
1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
1 4 2 3	2 4 3 1	3 4 2 1	4 3 2 1
1 4 3 2	2 4 1 3	3 4 1 2	4 3 1 2

There are 64 ways of arranging the numbers 1,2,3,4. (64 = 4 * 3 * 2 * 1)

It is very difficult to find the number of arrangement by actually arranging them if the number of items is more. E.g.: if we want to know how many ways we can arrange 1,2,3,4,5,6,7,8,9. It will be difficult to tell by actually arranging it. That is when permutation come to rescue.

Permutation formula gives us easy way to find number of arrangements for a given data.

Fundamental Principle of Counting

If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of different ways of :

(a) simultaneous occurrence of both the events in the given order is $m \times n$. This principle can be generalized for multiple events (known as multiplication principle).

(b) happening exactly one of the events is $m + n$ (known as addition principle).

Permutation with distinct objects

Let's assume the word "CARD" with distinct objects C, A, R,& D. How many 4 & 2 letter words can we form using letters of CARD, when repetition of letters is not allowed, also find the number of 4 letters word if repletion is allowed?

Solution:

Since we need to create 4 letters word, let's assume we have 4 bucket.

Case 1: 4 letters Repetition is not allowed

In this case, bucket 1 can have any of the 4 letters "C, A , R , D" . That is 4 choices.

Bucket 2 can have any of the remaining 3 letters as one letter is now inside bucket 1, that is 3 choices

Bucket 3 will can any of the remaining 2 letters that is 2 choices

Bucket 4 will have only 1 choice that is the remaining letter. Thus 1 choice.

If you multiple all these choices : 4* 3 * 2 *1 , you will get the number of possible 4 letter words with "C A R D", without repetition

Thus number of 4 letter possible words are : 4* 3 * 2 *1 = 24 words when repetition is not allowed.

Case 2: 4 letters Repetition is allowed

In this case, bucket 1 can have any of the 4 letters "C, A , R , D" . That is 4 choices.

Bucket 2 can also have any of the 4 letters, since repetition is allowed, that is 2 choices.

Bucket 3 can also have any of the 4 letters, since repetition is allowed, that is 2 choices.

Bucket 4 can also have any of the 4 letters, since repetition is allowed, that is 2 choices.

If you multiple all these choices: 4* 4 * 4 *4 , you will get the number of possible 4 letter words with "C A R D" with repetition.

Thus number of possible 4 letter words are : 4* 4 * 4 *4 = 256 words when repetition is allowed.

Case 3: 2 letters Repetition is not allowed

In this case, bucket 1 can have any of the 4 letters "C, A , R , D" . That is 4 choices.

Bucket 2 can have any of the remaining 3 letters as one letter is now inside bucket 1, that is 3 choices.

There is no bucket 3 & 4, as we have to find 2 letters word such as CA, CD, RD, RA etc.

If you multiple all these choices: 4* 3 , you will get the number of possible 2 letter words with "C A R D", without repetition.

Thus number of possible 2 letter words are : 4* 3 = 12 words when repetition is not allowed.

Case 4: 2 letters Repetition is allowed

In this case, bucket 1 can have any of the 4 letters "C, A , R , D" . That is 4 choices.

Bucket 2 can also have any of the 4 letters, since repetition is allowed, that is 2 choices.

There is no bucket 3 & 4, as we have to find 2 letters word such as CC, AA, CA, CD, RD, RA etc.

If you multiple all these choices: 4* 4 , you will get the number of possible 2 letter words with "C A R D" with repetition.

Thus number of possible 2 letter words are : 4* 4 = 16 words when repetition is allowed.

Thus we have seen 4 scenarios.

Find number of arrangement of compete set objects without repetition.

Find number of arrangement of compete set objects with repetition.

Find number of arrangement using smaller set objects without repetition.

Find number of arrangement using smaller set objects with repetition.

Let's create general formula for these scenarios.

Note that the scenario we have discussed has all the elements unique { C, A, R, D}, thus we can call it set.

Case 1: Without repetition & Permutation of whole set

Permutation = n * (n-1) * (n-2)…1

Case 2: With repetition & Permutation of whole set

Permutation = n* n * n …n times

Case 3: Without repetition & Permutation of sub set with r elements

Permutation = n * (n-1) * (n-2)…(n-r+1)

Case 4: With repetition & Permutation of sub set with r elements

Permutation = n* n * n …r times

Symbols used in Permutation

n! = n(n-1) (n-2) ……………….3.2.1.

$^nP_r = \frac{n!}{(n-r)!}$ = n * (n-1) * (n-2)…(n-r+1) For n>r

Numerical:

Find (i) 5! (ii) $\frac{7!}{5!}$ (iii) $^8P_5$

Solution:

5! = 5 * 4 * 3 * 2 * 1 = 120

$\frac{7!}{5!} = \frac{7*6*5*4*3*2*1}{5*4*3*2*1} = 42$

$^8P_5 \quad = \frac{8!}{(8-5)!}$

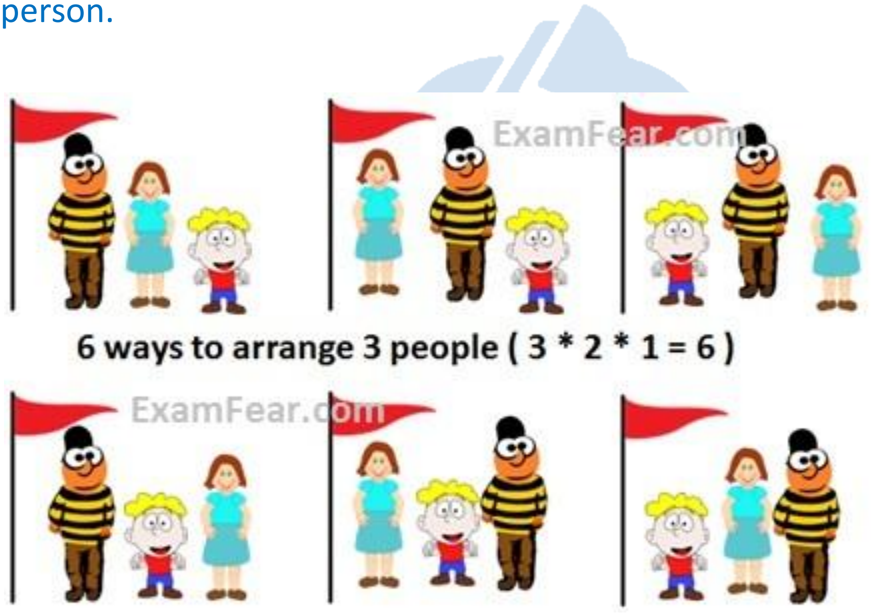
$= \frac{8!}{3!} = \frac{8*7*6*5*4*3*2*1}{3*2*1} = 6720$

Numerical:

How many ways 3 people can stand in a queue?

Solution:

In this case it is implicit that repetition is not allowed, as we can't repeat a person.



Theorem 1:

The number of permutations of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not repeat is ^nP_r.

Or $n (n – 1) (n – 2). . .(n – r + 1)$ Or $\frac{n!}{n-r}$

Numerical:

3 vacant seats, 6 people standing, how many ways they can sit.

Solution:

Let's solve this conventional way.

First Chair can be occupied by any of 6 persons, that is 6 choices

Second Chair can be occupied by any of 5 remaining persons, that is 5 choices

Third Chair can be occupied by any of 4 remaining persons, that is 4 choices

So, number of possible ways is 6 * 5 * 4 = 120

Now let's solve this using the formula. Number of possible arrangement is ^nP_r

In this case n = 6 & r=3.

Applying this in formula, we get Number of arrangement is $^6P_3$, that is $\frac{6!}{(6-3)!}$ = 120

Learnohub recommends the conventional logical way to solve Probability questions.

Theorem 2:

Number of permutations of n different objects taken r at a time, where repetition is allowed, is nr.

Numerical:

Find 3 digit numbers formed from (1,3,5,7,9) when repetition is allowed

Solution:

Let's solve using logical method. Since 3 digit numbers has to be formed, lets assume 3 buckets.

In this case, bucket 1 can have any of the 5 digits (1,3,5,7,9). That is 5 choices

Bucket 2 can also have any of the 5 digits, since repetition is allowed, that is 5 choices

Bucket 3 can also have any of the 4 digits, since repetition is allowed, that is 5 choices

So, total number of 3 digit numbers with repetition is 5 * 5 * 5 = 125

Let's solve using formula.

Total number of 3 digit numbers with repetition is nr , here n=5 & r =3

So, Total number of 3 digit numbers with repetition 53 = 125

Learnohub recommends the conventional logical way to solve Probability questions.

Permutation when all objects are not distinct

Sometimes we come up with scenarios where the objects are no distinct. For example, Let's take few green , blue, red & grey books. Now we have to find the number of ways we can arrange them.

Theorem 3:

The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p1! * p2! * p3! * \ldots pk!}$

Numerical:

Find the number of 9 letters word that can be formed using letters of ALLAHABAD.

Solution:

Let's solve logically first. Assume the repeated words to be different. Let ALLAHABAD be

$A_1L_1L_2A_2HA_3BA_4D$

Assume 9 buckets & each of these buckets has 9,8,7,6,5,4,3,2,1 choice. So total number of 9 letters word formed is 9*8*7*6*5*4*3*2*1 = 362880

But out of these words few words will be same as the there are repeated letters in ALLAHABAD.

A – 4 times, L 2 times. Thus we have to ignore these repeated words. This can be done by dividing the solution by 4! & 2!.

Thus number of 9 letters word formed using letters of ALLAHABAD is

$\frac{362880}{4! * 2!}$ =7560.

Now, let's solve using formula.

Number of 9 letters word formed using letters of $\frac{9!}{4! * 2!}$ = 7560

In some cases, we need to fix some of the objects are certain positions while finding the permutation. For example if we have to arrange 5 students in 4 seats with a condition that 1st seat is always occupied by a particular Student.

Numerical:

For word INDEPENDENCE. In how many of these arrangements, (i) do words start with P ii) do all vowels always occur together (iii) do vowels never occur together (iv) do words begin with I and end in P?

Solution:

There are 12 letters, of which N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different.

(i) If we fix the position of word P, we are left with 11 letters of which N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different.

Therefore, the required numbers of words starting with P are $\frac{11!}{3!*2!*4\,!}$ = 138600

(ii) If all vowels occur together, then lets consider vowels {EEEEI} as one entity. Thus number of object now are {(EEEEI),N, D, P, N, D, N,C) that is 8 objects with B repeating 3 times & D repeating 2 times.

These 8 objects, in which there are 3Ns and 2 Ds, can be rearranged in $\frac{8!}{3!\,2!}$ ways.

Also note that number of words formed using EEEEI will be $\frac{5!}{4!}$. As it has 5 words with E repeating 4 times. Corresponding to each of these arrangements, the 5 vowels E, E, E, E and I can be rearranged in $\frac{5!}{4!}$ ways.

Therefore, by multiplication principle the required number of arrangements is $\frac{8!}{3!2!} * \frac{5!}{4!}$ = 16800

(iii) To find number of words where vowels never occur together, let's find total number of possible words & then subtract the words where vowel occur together.

Total number of words with 12 letters where N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different is $\frac{12!}{3!*2!*4!}$ = 1663200

Total number of words with vowels not together = 1663200 – 16800 = 1646400

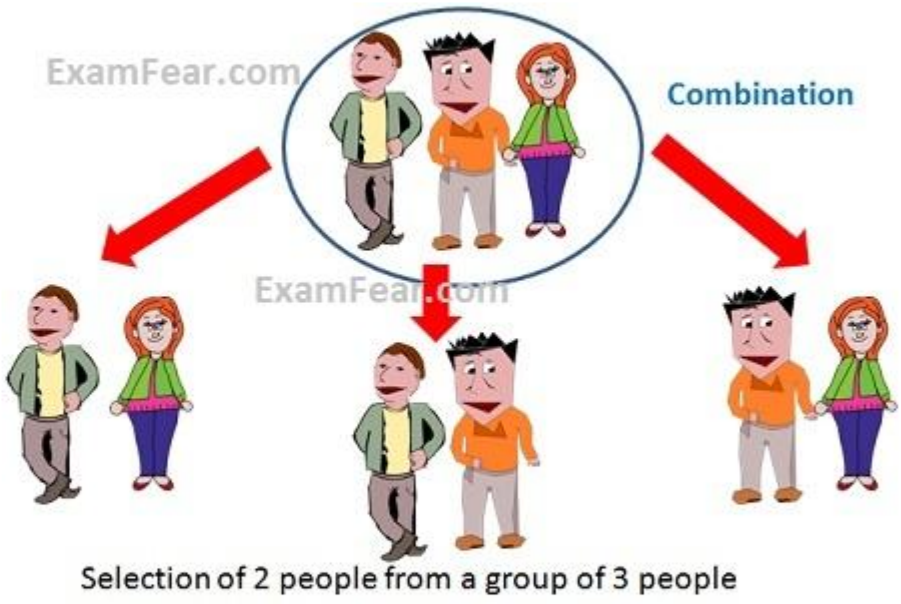
(iv) If we fix I and P at the extreme ends (I at the left end and P at the right end). We are left with 10 letters, where N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different is

Hence, the required number of arrangements is $\frac{10!}{3! * 2! * 4!}$ = 12600

Combination:

Combination means selection of things. Order of things has no importance.

E.g. Select 2 players from a pool of 3 players



If we have to select 80 students from a group from 500 students, then this manual approach will be nightmare, thus come Combination to rescue. It gives us a formula to find number of selections easily.

Number of ways of selecting r object from a group of n object is given by

$^nC_r = \frac{n!}{r! * (n-r)!}$

Other combination examples

Ten persons meet in a room and each shakes hand with all the others. How do we determine the number of handshakes? Here, order is not important.

Six points lie on a circle. How many chords can be drawn by joining these points pair wise?

If you have m shirts & n trousers, then you can have m x n different dress.

E.g. Let's assume you have 3 shirts & 2 trousers. Then possible dress is 3 * 2 = 6

Similarly if you have 2 school bags, 3 water bottles & 4 lunch box, then you can have 24 unique combinations

Permutation means arrangement of things. Order of things is important.

E.g. Number of ways in which 3 vacant distinct chairs to be occupied by 5 people.

Combination means selection of things. Order of things has no importance.

E.g. Select 3 people from a pool of 5 people

Theorem 5:

${}^{n}P_{r} = {}^{n}C_{r} * r!$ Where, $0 < r \leq n$

We can also say that ${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$

Useful Formula

$^nC_r + {}^nC_r - 1 = {}^{n+1}C_r$

$^nP_r = {}^nC_r * r!$ Where , $0 < r \leq n$.

$n! = n * (n-1)!$

Numerical:

If $^nC_9 = {}^nC_8$, Find $^nC_{16}$

Solution:

$^nC_9 = {}^nC_8$

Or $\frac{n!}{9!\,(n-9)!} = \frac{n!}{8!\,(n-8)!}$

Or $\frac{n!}{9*8!\,(n-9)!} = \frac{n!}{8!\,(n-8)!\,(n-9)!}$

Or n= 17

$^nC_{16} = {}^{17}C_{16} = 17$

Numerical:

What is number of ways to select 4 cards from a pack of 52 playing cards? In how many:

- four cards are of the same suit,
- four cards belong to four different suits,

- are face cards,
- two are red cards and two are black cards,
- cards are of the same colour?

Solution:

Out of total 52 cards, there are 12 Face cards (J, K , Q)

1) Ways to draw 4 cards of same suit = Ways to draw 4 cards of (diamond + Heart + Club + Space) suit

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

= 2860

2) Ways to draw 4 belonging to different suit =ways to draw 1 card each of Diamond, heart , Club & Spade suit

$= {}^{13}C_1 * {}^{13}C_1 * {}^{13}C_1 * {}^{13}C_1$

= 28561

3) Total number of face card (J, K, Q) = 4 * 3 = 12

Ways to draw 4 face card = ${}^{12}C_4$ = 495

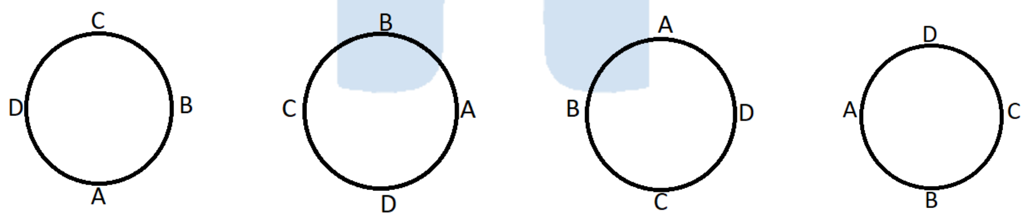
4) Way to draw two are red cards and two are black cards = $^{26}C_2 * ^{26}C_2$ = 105625

5) Ways to draw card of same color = ways to draw 4 red cards + ways to draw 4 black card

= $^{26}C_4 + ^{26}C_4$ = 29900

Circular Permutation:

Let's consider four persons A, B, C and D are sitting around a circular table. If all of them are shifted by one position in anti-clockwise direction, we get the following arrangements.



We observe that all the above shifted arrangements are same, because anti-clockwise order of A, B, C and D does not change.

So, if there are four objects, then for each circular arrangement, number of linear arrangements are four.

Therefore, the number of linear arrangements of n different objects = n×number of circular arrangements of n different objects.

Hence, number of circular arrangements of n different objects = $\frac{n!}{n}$ = (n-1)!

Note:

(i) If clockwise and anti-clockwise order of arrangement is considered same, then number of circular arrangements of n different objects = $\frac{(n-1)!}{2}$

(ii) The number of circular permutations of n different objects taking r at a time = $\frac{{}^{n}P_{r}}{r}$

(iii) If clockwise and anti-clockwise order of arrangement is considered same, then the number of circular permutations of n different objects taking r at a time = $\frac{{}^{n}P_{r}}{2r}$

Numerical:

In how many ways can 20 persons be seated around a table, if there are 10 seats ?

Solution:

In the case of circular table, clockwise and anti-clockwise orders are different.

Therefore, the required number of circular arrangements = $\frac{{}^{20}P_{10}}{10} = \frac{20!}{(10!)(10!)}$

<u>Restricted Selections:</u>

The number of selections of r objects out or n different objects, when

(a) k particular objects are always included = ${}^{n-k}C_{r-k}$

(b) k particular objects are never included = ${}^{n-k}C_r$

Numerical:

In how many ways can a cricket team of eleven players be chosen out of a batch of 16 players, if a particular player is always chosen.

Solution:

Since, a particular player is always chosen. It means that 11 – 1 = 10 players are to be selected out of the remaining 16 – 1 = 15 players.

Required number of ways = ${}^{15}C_{10}$ = ${}^{15}C_5$ = $\frac{15.14.13.12.11}{5.4.3.2.1}$ = 3003

Number of Divisors of N:

Every natural number N can always be put in the form

N = $(p_1)^{\alpha_1}.(p_2)^{\alpha_2}.(p_3)^{\alpha_3}.(p_4)^{\alpha_4}\ .\ .\ .\ .\ .\ (p_k)^{\alpha_k}$,

where $p_1, p_2, p_3, p_4, \ldots, p_k$ are distinct prime numbers and

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \ldots, \alpha_k$ are natural numbers

(i) Total number of divisors of N including 1 and N = $(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)$

(ii) Total number of divisors of N excluding 1 and N = $(\alpha_1+1)(\alpha_2+1)..(\alpha_k+1)-2$

(iii) Total number of divisors of N excluding either 1 or N

= $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots\ldots(\alpha_k+1) - 1$

(iv) Sum of all divisors = $(p_1{}^0 + p_1{}^1 + p_1{}^2 + p_1{}^3 + \ldots + p_1{}^{\alpha_1})(p_2{}^0 + p_2{}^1 + p_2{}^2 + p_2{}^3 + \ldots + p_2{}^{\alpha_2}) \ldots (p_k{}^0 + p_k{}^1 + p_k{}^2 + p_k{}^3 + \ldots + p_k{}^{\alpha_k})$

= $\left(\frac{1-(p_1)^{\alpha_1+1}}{1-p_1}\right)\left(\frac{1-(p_2)^{\alpha_2+1}}{1-p_2}\right)\ldots\ldots\left(\frac{1-(p_k)^{\alpha_k+1}}{1-p_k}\right)$

(v) Sum of Proper Divisors = Sum of all divisors – (N + 1)

(vi) Even divisors of N are possible only when $p_1 = 2$, otherwise there is no even divisor.

Total number of even divisors = $(\alpha_1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$

(vii) Odd divisors

Case I: If $p_1 = 2$, Total number of odd divisors = $(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$

Case II: If $p_1 \neq 2$, Total number of odd divisors = $(\alpha_1+1)(\alpha_2+1)(\alpha_3+1)\ldots(\alpha_k+1)$

(viii) Number of ways in which N can be resolved as a product of two factors :

Case I: If N is not a perfect square, Total number = $\frac{1}{2}(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)$

Case II: If N is a perfect square, Total number = $\frac{1}{2}\{(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)+1\}$

Numerical:

Find the total number of divisors of the number 1008.

Solution:

$1008 = 2^4 \times 3^2 \times 7^1$

Total number of divisors = (4+1)(2+1)(1+1) = 30

Division of Objects into Groups:

(a) Number of ways in which $(x_1+x_2+x_3+......+x_n)$ distinct objects can be divided into n unequal groups containing $x_1, x_2, x_3, ..., x_n$ objects is $\frac{(x_1+x_2+x_3+\cdots...+x_n)!}{x_1!\,x_2!\,x_3!......x_n!}$

(b) Number of ways to distribute $(x_1+x_2+x_3+......+x_n)$ distinct objects among n persons having $x_1, x_2, x_3, ..., x_n$ objects is $\frac{(x_1+x_2+x_3+\cdots...+x_n)!}{x_1!\,x_2!\,x_3!......x_n!} \times n!$

(c) Number of ways in which (mn) distinct objects can be divided equally into m groups, each containing n objects and

(i) If order of groups is not important, is = $\left(\frac{(mn)!}{(n!)^m}\right) \times \frac{1}{m!}$

(ii) If order of groups is important, is = $\left(\left(\frac{(mn)!}{(n!)^m}\right) \times \frac{1}{m!}\right) \times m! = \left(\frac{(mn)!}{(n!)^m}\right)$

Numerical:

In how many ways can a pack of 52 cards be

(i) divided into four groups of 13 cards each ?

(ii) distributed equally among four players in order ?

(iii) divided into four sets, three of them having 16 cards each and fourth having four cards ?

Solution:

(i) Here order of group is not important, then the number of ways in which 52 different cards can be divided equally into 4 groups = $\frac{52!}{(13!)^4 4!}$

(ii) Here order of group is important, then the number of ways in which 52 different cards can be divided equally into 4 groups = $\frac{52!}{(13!)^4 4!} \times 4! = \frac{52!}{(13!)^4}$

(iii) Here, firstly we need to divide 52 cards into two sets containing 4 and 48 cards. And then, 48 cards is divided equally in three sets each containing 17 cards.

Required number of ways = $\frac{52!}{4!\,48!} \times \frac{48!}{(16!)^3 3!} = \frac{52!}{(16!)^3 4!\,3!}$

Rank in a Dictionary:

If the letters of a word is arranged as in a dictionary, then we are asked to find the rank of that word.

We will learn to find Rank of a word with the help of the following example.

Numerical:

If the letters of the word SURITI are arranged as in an English Dictionary, find the rank of SURITI.

Solution:

The letters in alphabetical order are I, I, R, S, T, U

Number of words beginning with I = 5! = 120

Number of words beginning with R = $\frac{5!}{2!}$ = 60

Number of words beginning with SI = 4! = 24

Number of words beginning with SR = $\frac{4!}{2!}$ = 12

Number of words beginning with ST = $\frac{4!}{2!}$ = 12

Number of words beginning with SUI = 3! = 6

Number of words beginning with SURII = 1! = 1

Number of words beginning with SURITI = 1

Rank of the word SURITI = 120+60+24+12+12+6+1+1 = 236

Arrangement and Derangement:

(a) The number of ways in which n different objects can be arranged into r different groups is r(r+1)(r+2)(r+3) . . . (r+n-1) or $n!.{}^{n-1}C_{r-1}$

(b) The number of ways in which n different objects can be distributed into r different groups is $r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n - . . . + (-1)^n.{}^rC_{r-1}$

Or $\sum_{p=0}^{r}(-1)^p.{}^rC_p(r-p)^n$

Or Coefficient of x^n in $n!(e^x - 1)^r$

(c) Case I: If blank groups are not allowed,

The number of ways in which n identical objects can be distributed into r different groups = ${}^{n-1}C_{r-1}$

Case II: If blank groups are allowed,

The number of ways in which n identical objects can be distributed into r different groups = ${}^{n+r-1}C_{r-1}$

Derangement: If ‘n’ objects are to be placed at ‘n’ specific places but none of them occupy its original space.

Number of Derangements, D_n = n!$\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right]$

Important Results: D_1 = 0, D_2 = 1, D_3 = 2, D_4 = 9, D_5 = 44, D_6 = 265

Numerical:

In how many ways 6 different balls can be arranged into 3 different boxes so that no box remains empty ?

Solution:

Required number of ways = (6!) $.{}^{6-1}C_{3-1} = (6!)\ .{}^{5}C_2$

$= 720(10)$

$= 7200$

Numerical:

In how many ways can 25 identical mangoes be distributed among 3 boys?

Solution:

Here each boys can receive any number of mangoes.

So, total number of ways = ${}^{25+3-1}C_{3-1}$ = ${}^{27}C_2$ = 351

Numerical:

Find the number of ways to put 4 letters in 4 addressed envelopes so that all goes in wrong envelopes.

Solution:

Number of ways such that all 4 goes in wrong envelopes,

$D_4 = 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = (24)\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right] = 12 - 4 + 1 = 9$

Multinomial Theorem:

(i) If there are l objects of one kind, m objects of second kind, n objects of third kind and so on, then the number of ways of choosing r objects out of these objects is the coefficient of x^r in the expansion of

$(1+x+x^2 + x^3 + \cdots + x^l)\ (1+x+x^2 + x^3 + \cdots + x^m)\ (1+x+x^2 + \cdots + x^n) \ldots..$

(ii) If there are l objects of one kind, m objects of second kind, n objects of third kind and so on, then the number of possible arrangements/permutations of r objects out of these objects is the coefficient of x^r in the expansion of

$r!\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^l}{l!}\right)\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^m}{m!}\right)\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}\right) \ldots.$

Numerical:

Find the number of combinations and permutations of 4 letters taken from the word EXAMINATION.

Solution:

There are 11 letters – A, A, I , I, N, N, E, X, M, T, O

Then, number of combinations

= coefficient of x^4 in $(1+x+x^2)^3(1+x)^5$

= coefficient of x^4 in $\{(1+x)^3+x^6+3(1+x)^2x^2+3(1+x)x^4\}(1+x)^5$

= coefficient of x^4 in $\{(1+x)^8+x^6(1+x)^5+3x^2(1+x)^7+3x^4(1+x)^6\}$

= $^8C_4 + 0 + 3.^7C_2 + 3$

= 70 + 63 + 3

= 136

Number of permutations

= coefficient of x^4 in $4!\left(1+\frac{x}{1!}+\frac{x^2}{2!}\right)^3\left(1+\frac{x}{1!}\right)^5$

= coefficient of x^4 in $4!\left(1+x+\frac{x^2}{2}\right)^3(1+x)^5$

= coefficient of x^4 in $4!\left\{(1+x)^3+3(1+x)^2\frac{x^2}{2}+3(1+x)\frac{x^4}{4}+\frac{x^6}{8}\right\}(1+x)^5$

= coefficient of x^4 in $4!\left\{(1+x)^8+\frac{3}{2}x^2(1+x)^7+\frac{3}{4}x^4(1+x)^6+x^6(1+x)^5\right\}$

= $4!\left\{^8C_4+\frac{3}{2}.^7C_2+\frac{3}{4}+0\right\}$

= $24\left(70+\frac{63}{2}+\frac{3}{4}\right)$

= 1680 + 756 + 18

= 2454

Geometrical Problems:

If there are n points in a plane :

(i) Number of different lines obtained using these n points = ${}^{n}C_2$

(ii) Number of points of intersections of the lines joining these n points = ${}^{p}C_2$, where p = ${}^{n}C_2$

(iii) Number of different triangles obtained using these n points = ${}^{n}C_3$

(iv) If n points are the vertices of a polygon, then total number of diagonals = ${}^{n}C_2 - n = \frac{n(n-3)}{2}$

Numerical:

In a polygon the number of diagonals is 65. Find the number of sides of the polygon.

Solution:

Let the number of sides of the polygon be n.

Then number of diagonals = $\frac{n(n-3)}{2}$ = 65

n(n - 3) = 130 = 13×10

n = 13.

****